

Exercise 12

A point P in the first quadrant lies on the graph of the function $f(x) = \sqrt{x}$. Express the coordinates of P as functions of the slope of the line joining P to the origin.

Solution

A point on the curve $f(x) = \sqrt{x}$ is

$$(x, \sqrt{x}). \quad (1)$$

The equation of a line going through the origin is

$$y = mx.$$

The y -coordinate is \sqrt{x} units high.

$$\sqrt{x} = mx$$

Solve for x by dividing both sides by x .

$$\frac{1}{\sqrt{x}} = m$$

Then invert both sides.

$$\sqrt{x} = \frac{1}{m}$$

Then square both sides.

$$x = \frac{1}{m^2}$$

Therefore, plugging this formula into (1), the coordinates of P in terms of the slope of the line joining P to the origin are

$$\left(\frac{1}{m^2}, \frac{1}{m} \right).$$